CHAPTER 4

DEFINITION OF THE SYSTEM MODEL

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4.1 INTRODUCTION

This chapter is devoted to the definition of a system model tailored towards UML. The hierarchy of theories that compose the system model is stepwise introduced. These theories are combined into a theory of sophisticated state transition systems. The semantics of a word in a UML sublanguage, i.e., a diagram, can then be defined by a set of such transition systems. Given two or more actual diagrams, possibly forming a complete UML model, the semantics of them together is defined by the intersection of their translations into the system model. In other words, consistency of a model is defined by non-empty intersection of the sets containing the transition systems that implement the diagrams individually.

UML2 Semantics and Applications. By Kevin Lano

[BCGR09a] M. Broy, M. V. Cengarle, H. Grönniger, B. Rumpe
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Definition 4.2.1 (This is a definition)

DefinitionName

introduction of new elements (sets, functions, ...)

Notation:
additional notational abbreviations (optional)

definition of properties that hold

informal, textual explanation (optional)

The system model supports underspecification in two manners. On the one hand, the fact that the semantics of a single UML diagram or of a complete UML model is not univocal is a form of underspecification. On the other hand, at the metalevel, the system model can be further constrained in such a way that the ambiguity inherent to the language is reduced or even eliminated. The latter ambiguities are called variation points, and the choice of a particular variant reduces the range of possibilities.

The former chapter motivates and explains the system model that is introduced below. The system model in this chapter is a simplification of the one presented in [1]. In that work, a number of variation points are also presented. These include, among others, records and Cartesian products within the type system of the system model, subclassing observing structure, objects as values, locations and reference types, qualified and ordered binary associations, active vs. passive objects, single vs multithreaded computation.

This chapter is organized as follows. Section 4.3 contains the definition of the structural part of the system model. Sections 4.4 and 4.5 contain the control and communication related definition definitions which form the basis to describe the state of a system in Section 4.6. Two variants of state transitions systems are introduced to define object behaviour in Sections 4.7 (event-based) and 4.8 (timed).

4.2 NOTATIONAL CONVENTIONS

This section contains the conventions used in order to structure the mathematical theories that constitute the system model.

Definitions, presented as shown in 4.2.1, usually contribute new elements to the system model and/or add constraints to already existing ones. Noteworthy derived properties following from a definition are stated as Lemmas, that are presented in way similar to that of definitions.

In order to simplify the notation, if a formula contains a symbol whose value is irrelevant for the purpose of the fomula, the symbol is replaced by a wildcard $*$. For instance, $\forall a : P(a,*,*)$ stands for $\forall a : \exists y,z : P(a,y,z)$ where the variables y,z are unused within the predicate P and are existentially quantified at the innermost level. Also a number of container structures is used, such as $\mathbb{P}(.)$ for powerset, $\mathbb{P}_f(.)$ for the set of finite subset of a given one, $List(.)$, $Stack(.)$, and $Buffer(.)$ for the

Definition 4.3.1 (Types and values)

*Type*1

$UTYPE$
$UVAL$
$CAR : UTYPE \rightarrow \mathbb{P}(UVAL)$

$\forall u \in UTYPE : CAR(u) \neq \varnothing$

UTYPE is the universe of type names.
UVAL is the universe of values.
CAR maps type names to non-empty carrier sets; carrier sets need not be disjoint.

usual constructs. These structures are defined in mathematical terms with appropriate manipulation and selection functions. For details on these basics, the reader may consult report [1].

4.3 STATIC PART OF THE SYSTEM MODEL

The static part of the system model contains the unalterable information of the intended systems. The static part is composed of, among other things, some universes of elements that are assumed given and not fully described here. Properties of and relationships between those universes may be furthermore assumed. Universes are for instance, the universe of type names *UTYPE*, the universe of values *UVAL*, a relation *CAR* that associates a set of values to each type name (see Definition 4.3.1), the universe of class names *UCLASS*, and the universe of object identifiers *UOID* (see Definition 4.3.5). The primitive concept of name is not futher prescribed.

4.3.1 Type Names and Carrier Sets

A type name identifies a carrier set which contains simple or complex data elements called members or values of (or associated with) the type name. The universe of all type names is denoted by *UTYPE*. Members of all type names are gathered in the universe *UVAL* of values; see Definition 4.3.1.

Any $T \in UTYPE$ is a type *name*, not a type, but may be referred to as type T. In particular, the types of the system model explicitly encode UML types, i.e., deep embedding of types is used.

The above definitions leave open quite a number of possibilities to characterize types. [1] shows a few examples, which are not formal parts of the system model. For example, we may wish to express that integer and float are type names, i.e., $Int, Float \in UTYPE$, integer and floating point values are values in the system model $\mathbb{R} \subseteq UVAL$, and that integer values are also floats: $CAR(Float) = \mathbb{R}$ and $CAR(Int) = \mathbb{Z} \subseteq \mathbb{R}$.

Definition 4.3.2 (Basic types)

BoolInt

$Bool, Int \in UTYPE$
$true, false \in UVAL$

$CAR(Bool) = \{true, false\}$
$true \neq false$
$CAR(Int) = \mathbb{Z} \subseteq UVAL$

UTYPE contains at least the type names Bool and Int.
UVAL contains at least Boolean and integer values.

Definition 4.3.3 (Basic type Void)

Void

$Void \in UTYPE$
$void \in UVAL$

$CAR(Void) = \{void\}$

---

void can be used to e.g. return control without an actual return value.

The value *void* is usually needed for giving semantics to procedures or methods with no return value. This is customary in the semantics of programming languages.

4.3.2 Basic Type Names and Type Name Constructors

Basic type names for basic values such as Boolean and integer values (see Definition 4.3.2) are by default given, together with their typical operations like logical connectives and arithmetic operators (not detailed here). The carrier set associated with *Void*, a further basic type name, is a singleton; see Definition 4.3.3.

4.3.3 Variables

The notion of variable (see Definition 4.3.4) permits the encoding of object attributes, method parameters and method local variables. Each variable name has a unique type name associated.

4.3.4 Class Names and Objects, Subclass Relation

Definition 4.3.5 introduces the universes *UCLASS* of class names, *UOID* of object identifiers, and *INSTANCE* of instances. A class name is associated with a finite set of attributes, which simply are variables. Each class name is moreover associated with a set of object identifiers. By use of the association mechanism (see Section 4.3.7),

Definition 4.3.4 (Variables, attributes, parameters)

Variable

$UVAR$
$vtype : UVAR \to UTYPE$
$vsort : UVAR \to \mathbb{P}(UVAL)$
$VarAssign = (v : UVAR \rightharpoonup vsort(v))$

Notation:
$a : T$ denotes "a is a variable of type T", i.e., $vtype(a) = T$.

$\forall\, v \in UVAR : vsort(v) = CAR(vtype(v)) \wedge \forall\, val \in VarAssign : val(v) \in vsort(v)$

UVAR is the universe of all variable names, each with a unique type associated.
VarAssign is the set of all total and partial assignments of values for variables.

class names can be related to each other. Also methods can be associated with class names; see Definition 4.4.2.

Except for the object identifier *Nil* (see Definition 4.3.7), there is a bijection between the universes *UOID* and *INSTANCE*. Thus, besides *Nil*, there are no dangling references. As a consequence each object belongs exactly to one class,[2] and this does not vary over time, whereas the object value can vary and dereferencing from an object identifier is state dependent. (In particular, structurally equivalent classes are distinguished.)

More precisely, *UOID* contains references to all possible objects and, in a similar way, *INSTANCE* contains all possible objects. These sets are usually infinite because they represent the possible existence of objects. Furthermore, *INSTANCE* contains all object values thus describing many different object values with the same identifier. At each point of time only a finite subset of objects will actually exist in the data store (see Section 4.3.6 below) and there will be at most one instance for any identifier.

Definition 4.3.6 introduces the mechanisms for accessing the attributes of an object. A distinguished term is *this*, which can be treated as were it an attribute though it is not (and thus does not appear in $attr(C)$). In particular, no type name is associated with *this*, thus a number of conceptually difficulties are avoided like recursive type definitions.

Definition 4.3.7 introduces the special identifier *Nil* and constrains *UOID* to exactly consist of object identifiers and *INSTANCE* of objects only.

4.3.5 Subclass Relation

The *subclass* (or *inheritance*) relation *sub* is introduced in Definition 4.3.8. There, also a type name contructor is introduced, that associates a type name with each class name; this type name collects in its carrier set all the object identifiers associated

[2] Polymorphism is introduced below in Section 4.3.5.

Definition 4.3.5 (Classes and instances)

Class

$UCLASS$
$UOID$
$INSTANCE$
$attr : UCLASS \to \mathbb{P}_f(UVAR)$
$oids : UCLASS \to \mathbb{P}(UOID)$
$objects : UCLASS \to \mathbb{P}(INSTANCE)$
$objects : UOID \to \mathbb{P}(INSTANCE)$
$classOf : INSTANCE \to UCLASS$
$classOf : UOID \rightharpoonup UCLASS$

$\forall\, oid \in oids(C) : classOf(oid) = C \wedge$
$\qquad objects(oid) = \{(oid, r) \mid r \in VarAssign \wedge attr(r) = attr(C)\}$
$\forall\, o \in objects(C) : classOf(o) = C$

$UCLASS$ is the universe of class names.
$attr$ assigns a finite set of attributes to each class name.
$UOID$ is the universe of object identifiers.
$INSTANCE$ is the universe of objects of the form $o = (oid, r)$ where oid is an object identifier and r a variable assignment for the attributes of the class name associated with oid.
$oids$ assigns a set of object identifiers to a class name.
$classOf$ enforces uniqueness of the class (name) associated with each object and with each identifier.

Definition 4.3.6 (Attribute access)

Attribute

$this : INSTANCE \to UOID$
$getAttr : INSTANCE \times UVAR \rightharpoonup UVAL$
$attr : INSTANCE \to \mathbb{P}_f(UVAR)$
$attr : UOID \to \mathbb{P}_f(UVAR)$

Notation:
$o.this$ is shorthand for $this(o)$
$o.a$ is shorthand for $getAttr(o, a)$

$\forall\, o, (oid, r) \in INSTANCE :$
$\quad this((oid, r)) = oid$
$\quad getAttr((oid, r), a) = r.a$
$\quad attr(oid) = attr(classOf(oid))$
$\quad attr(o) = attr(classOf(o))$

$o.this$ is written in the spirit of attribute selection, but treated differently: $this$ is not an actual attribute of the class.

Definition 4.3.7 (Introduction of *Nil*)

Nil

$Nil \in UOID$

$\forall C \in UCLASS : Nil \notin oids(C)$
$\forall o \in INSTANCE : o.this \neq Nil$
$UOID = \{Nil\} \cup \bigcup_{C \in UCLASS} oids(C)$
$INSTANCE = \bigcup_{C \in UCLASS} objects(C)$

Nil is a distinguished object identifier, the only one not associated to any class or any object. *UOID* and *INSTANCE* only consist of object identifiers and of objects, respectively.

Definition 4.3.8 (Subclass Relation)

Subclassing

$sub \subseteq UCLASS \times UCLASS$
$\cdot^{\&} : UCLASS \rightarrow UTYPE$

$UOID \subseteq UVAL$

$transitive(sub) \wedge reflexive(sub)$
$\forall C \in UCLASS : CAR(C^{\&}) = \{Nil\} \cup \bigcup_{C_1 \; sub \; C} oids(C_1)$

sub is the transitive and reflexive subclass relation.
The carrier set associated with type name $C^{\&}$ contains all object identifiers that belong to the carrier set of class name *C* or any of its subclass names.

with class name or any of its subclass names. Therefore, object identifiers are values, i.e., $UOID \subseteq UVAL$.

Thus, the subclass relation allows a precise definition of the type of a class: the object identifiers associated with the class and any of its subclasses belong to the carrier of the type assigned to the class.

Definition 4.3.8 leaves a number of questions open and allows thus further refinement. For instance the binary relation *sub* is not enforced to be antisymmetric (although this is the case in any implementation language today). Furthermore, subclassing is not based on a structural definition: the sets of attributes of two classes may be in the subset relation, and nevertheless the classes be unrelated by *sub*.

The technique of defining *sub* as a subset relation on object identifiers instead of objects permits a great simplification on the type system within the system model. Furthermore, it allows the redefinition of attribute structures in subclasses without an otherwise necessary loss of the substitution principle.

Definition 4.3.9 (The data store)

*DataStore*1

$$DataStore \subseteq (UOID \rightarrow INSTANCE)$$
$$oids : DataStore \rightarrow \mathbb{P}(UOID)$$

$$\forall\, ds \in DataStore : oids(ds) = \mathrm{dom}(ds)$$
$$\forall\, o \in UOID, ds \in DataStore : ds(o).this = o$$

DataStore is the set of all data stores or possible snapshot values.
$oids(ds)$ is the set of existing objects in given a data store ds.

Definition 4.3.10 (DataStore Infrastructure)

DataStore

$$val : DataStore \times UOID \times UVAR \rightharpoonup UVAL$$
$$setval : DataStore \times UOID \times UVAR \times UVAL \rightharpoonup DataStore$$
$$addobj : DataStore \times INSTANCE \rightarrow DataStore$$

Notation:
$ds(oid.at)$ is shorthand for $val(ds, oid, at)$
$ds[oid.at = v]$ is shorthand for $setval(oid, at, v)$

$$\forall\, ds \in DataStore, oid \in oids(ds), at \in attr(oid), v \in CAR(vtype(at))) :$$
$$val(ds, oid, at) = ds(oid).at$$
$$setval(ds, oid, at, v) = ds \oplus [oid = (oid, \pi_2(ds(oid)) \oplus [at = v])]$$
$$o.this \notin oids(ds) \Rightarrow addobj(ds, o) = ds \oplus [o.this = o]$$

val retrieves the value for a given object and attribute.
setval updates a value for a given object and attribute.
addobj adds a new object.

4.3.6 Data Store Structure

Intuitively, a data store is a snapshot of the data state of a running system. Definition 4.3.9 introduces data stores as functions assigning objects to object identifiers. Any such function assigns an object o to an object identifier *oid* only if this is the identifier of that object, which can be retrieved using $o.this$.

A number of convenient retrieval and update functions for data stores is given in Definition 4.3.10. They basically deal with lookup and change of attribute values as well as "creating" a new object in the store.

Various restrictions on the use of retrieval and update functions apply. They involve the use of values of appropriate type, attributes that actually exist in a class, etc. However, we refrain from defining these restrictions here.

Definition 4.3.11 (Basic definitions for associations)

Association

$UASSOC$

$classes : UASSOC \to List(UCLASS)$

$extraVals : UASSOC \to \mathbb{P}(UVAL)$

$relOf : UASSOC \times DataStore \to \mathbb{P}(UVAL \times UVAL)$

$$\forall R \in UASSOC, C_i \in UCLASS(i = 1, \ldots, n), ds \in DataStore :$$
$$classes(R) = [C_1, \ldots, C_n] \Rightarrow$$
$$relOf(R, ds) \subseteq (CAR(C_1^{\&}) \times \ldots \times CAR(C_i^{\&})) \times extraVals(R)$$

UASSOC is the universe of association names.
classes returns the list of class names related by a given association name.
extraVals of a given association name is the set of further values that accompany with the association name.
relOf is the retrieval function to derive the actual links for an n-ary association based on the current store.

4.3.7 Associations

Definition 4.3.11 introduces the universe *UASSOC* of association names. The function *classes* associates a list of class names to each association name; given an association name R, $classes(R) = [C_1, \ldots, C_n]$ is sometimes called the signature of R. *classes* assigns a list, and not a set, of class names to an association name. The order of the classes is relevant as in, for example, a self-association like "parent-child".

Additional values that accompany an association name can be retrieved using the function *extraVals*. Addtional values permit qualified associations by using one (or more) of them as qualifier. They moreover allow non-unique associations: by introducing a value as distinguishing flag, a tuple can be duplicated.

Given a system snapshot, i.e., a data store, the relation retrieval function *relOf* returns the tuples that constitute the (instantiation of) the association name, each one together with the additional values *extraVals*, in that data store. These tuples contain values of the corresponding types; these types, in turn, are obtained from the class names in the signature of the association name using the type name constructor $\cdot^{\&}$. This means, the tuples of an instantiation of the association name may include objects identifiers whose *classOf* is a subclass of the corresponding class in the signature of the association name.

Restrictions on the changeability of an association, as UML class diagrams may impose, can only be observed or checked when two consecutive *DataStore*s are compared. This means, the semantics of a class diagram cannot be completely defined using only one snapshot of the *DataStore*.

Variation Point: Simple Associations Only Variation Point 4.3.12 is not formally part of the system model but shows how to refine it by adding further constraints. Each of the constraints may be imposed individually.

Variation Point 4.3.12 (Simplified associations)

$[SimplAssociation]$

$$\forall R \in UASSOC, ds \in DataStore : \#relOf(R, ds) \in \mathbb{N}$$

$$\forall R \in UASSOC : \#extraVals(R) = 1$$

$$\forall R \in UASSOC : \#classes(R) = 2$$

$$\forall R \in UASSOC, ds \in DataStore, oid \in UOID : \\ \#\{(oid_1, oid_2, x) \in relOf(R, ds) \mid oid = oid_1\} = 1$$

These constraints restrict the instantiations of association names to finite sets of tuples, disallow additional values for association names, enforce association names to be binary, or all association names to have multiplicity *1-to-** (which includes *1-to-1*), but not **-to-**.

Other variation points may define binary, qualified, and ordered association names as well as realization techniques for them.

4.4 CONTROL PART OF THE SYSTEM MODEL

Once defined the data part, this and the following section focus on the control part of the system model. The control part defines the structure used to store control information. Roughly speaking, this structure is divided into a control store and an event store. The control store contains all the information needed to determine the state of the system during computation.

That is, in addition to its data store as introduced in Definition 4.3.10, a state machine of the system model has a control store. This store contains information about the behaviour of the intended system and is used by the state machine in order to decide which transition to perform next. A control store consists of:

- a stack of method/operation calls, each with its arguments and local variables,
- the progress of the running program (e.g., a program counter), and
- possibly information about one or more threads.

In any setting, be it distributed or not, any state machine of the system model also has to deal with receiving and sending of events that trigger activities in objects. General events, such as “message arrived” or “timeout”, must be handled by any object. These events are put into an event store, which consists of an event buffer for each object where handling of events is managed. The event store, which is the last constituent of the state an object, is defined in Section 4.5.

Definition 4.4.1 (Definition of operations)

Operation

$$
\begin{array}{l}
UOPN \\
UOMNAME \\
nameOf : UOPN \to UOMNAME \\
classOf : UOPN \to UCLASS \\
parTypes : UOPN \to List(UTYPE) \\
params : UOPN \to \mathbb{P}(List(UVAL)) \\
resType : UOPN \to UTYPE \\
\hline
\forall\, op \in UOPN : parTypes(op) = [T_1, \ldots, T_n] \Rightarrow \\
\quad params(op) = \{[v_1, \ldots, v_n] : v_i \in CAR(T_i)(i = 1, \ldots, n)\}
\end{array}
$$

UOPN is the universe of operations.
UOMNAME the universe of operation (or method) names.
nameOf returns the name of a given operation.
classOf returns the class to which a given operation belongs.
parTypes returns the list of types of the parameters of a given operation.
params returns all possible arguments of the given operation.
resType returns the result type of the given operation.

4.4.1 Operations

Objects are accessed through their methods/operations. Here, the term operation refers to the signature (or head) whereas the term method refers (also) to the implementation (or body). Operations can be called and they may provide a return value as given by the corresponding implementation. Each operation has a name and a signature (which includes arguments and a return value that may be of type *Void*).

Definition 4.4.1 specifies signatures, which consist of a (possibly empty) list of types for parameters and a type for the return value. Note that parameter names are not present in the signature; parameter names are only part of the implementation. For each operation, its signature and its implementation, as well as the class it belongs to, are uniquely specified.

The subclassing mechanism lets subclasses inherit operations from their superclasses. This means, subclassing imposes a constraint on signatures and, in many languages, also a constraint on the promised behaviour of its related classes. Definition 4.4.2 relates operations in super-/subclasses in the co/contra-variant way (see e.g. [7]). The operation in the subclass may accept a superset of parameter values and may return a subset of return values, compared to the possible values of the superclass operation. In this way, the subclass operation can safely substitute the superclass operation.

Definition 4.4.2, although rather general, needs not to hold in all object-oriented languages. In particular, languages such as Smalltalk exhibiting “Message not un-

Definition 4.4.2 (Definition of type safety on operations)

TypeSafeOps

$$\forall op_1 \in UOPN, c \in UCLASS : c \, sub \, classOf(op_1) \Rightarrow \\ \exists op_2 \in UOPN : classOf(op_2) = c \wedge \\ nameOf(op_1) = nameOf(op_2) \wedge \\ CAR(resType(op_1)) \supseteq CAR(resType(op_2)) \wedge \\ params(op_1) \subseteq params(op_2)$$

Any class type-safely inherits operations from any of its superclasses.

derstood" errors to which a program can react, do not enforce this type safety requirement.

In the system model, operations have exactly one return value. Multiple return values, however, can be encoded, for instance by packing them in a class or record.

4.4.2 Methods

The term operation only refers to the signature whereas the term method refers (also) to the implementation of an operation. Methods, thus, have a signature and an internal implementation. The signature of a method consists of a list of parameter names with their types. Projected on the list of types, this list coincides with the parameter type list of the associated operation(s).

To provide all information necessary for a detailed understanding of method interactions, a binding mechanism between arguments and corresponding formal parameters is needed, as well as a store for local variables and an abstract notion of a program counter, as given in Definition 4.4.3. Furthermore, a method is equipped with the class name to which it belongs and where it is implemented. Note that *localsOf* and *parOf* result in variable assignments that contain mappings of variables to appropriate values. For convenience, parameters, attributes and local variables of a method are assumed disjoint (which is allowed by syntactic resolution).

The concepts of method (implementation) and of operation (signature) are fully decoupled, which allows their mutually independent description. However, there usually is a strong link between methods and operations: A method can only implement operations with compatible signatures. Nevertheless, as implementations can be inherited, multiple operations can refer to the same method as its implementation. In this way, on the one hand the operation signature can be adapted (e.g., made more specific) without changing the implementations, and on the other the implementation can be redefined using a new method in a subclass. Definition 4.4.4 describes this relation through a function *impl* that associates a method with a signature; if the class can be instantiated, all operations of that class need to have implementations.

The signature $params(op)$ of an operation op is a set of lists of values, whereas the parameter list $parNames(m)$ of the corresponding method implementation $m = impl(op)$ is a single list of variables.

Definition 4.4.3 (Definition of methods)

*Method*1

$$
\begin{array}{l}
UMETH \\
UPC \\
nameOf : UMETH \to UOMNAME \\
definedIn : UMETH \to UCLASS \\
parNames, localNames : UMETH \to List(UVAR) \\
parOf : m : UMETH \to \mathbb{P}(VarAssign|_{set(parNames(m))}) \\
localsOf : m : UMETH \to \mathbb{P}(VarAssign|_{set(localNames(m))}) \\
resType : UMETH \to UTYPE \\
pcOf : UMETH \to \mathbb{P}_f(UPC)
\end{array}
$$

$$
\begin{array}{l}
\forall\, m \in UMETH, v \in UVAR, val \in parOf(m) : \\
\quad v \in dom(val) \Leftrightarrow v \in set(parNames(m)) \\
\forall\, m \in UMETH, v \in UVAR, val \in localsOf(m) : \\
\quad v \in dom(val) \Leftrightarrow v \in set(localsNames(m)) \\
\quad parNames(m) \cap localNames(m) = \varnothing \\
\quad parNames(m) \cap attr(definedIn(m)) = \varnothing \\
\quad localsNames(m) \cap attr(definedIn(m)) = \varnothing
\end{array}
$$

UMETH is the universe of methods.
UPC is the universe of program counter values.
Given a method, *definedIn* returns the class to which the method (implementation) belongs (and where it was defined).
parNames returns the formal parameter variables of a given method.
localNames returns local variables of a given method.
parOf and *localsOf* return sets of variable assignments of formal parameters and of local variables, respectively, of a given method.
resType returns the result type of a given method.
pcOf is (finite) the set of possible program counter values of a method.
Pairwise disjointedness of *parNames*, *localNames* and *attr* is assumed for convenience.

Definition 4.4.4 (Relationship between method and operation)

Method

$$impl : UOPN \rightharpoonup UMETH$$

$$\begin{aligned}
&\forall\, op \in UOPN : m = impl(op) \Rightarrow \\
&\quad nameOf(m) = nameOf(op) \wedge \\
&\quad classOf(op)\ sub\ definedIn(m) \wedge \\
&\quad CAR(resType(m)) \subseteq CAR(resType(op)) \wedge \\
&\quad n = length(parNames(m)) \Rightarrow \\
&\qquad \{[val(parNames(m)_1), \ldots, val(parNames(m)_n)] : val \in parOf(m)\} \\
&\qquad\quad \supseteq params(op) \\
&\forall\, c \in UCLASS, op \in UOPN : oids(c) \neq \varnothing \wedge classOf(op) = c \Rightarrow \\
&\quad op \in dom(impl)
\end{aligned}$$

impl assigns a method implementation to each operation.

4.4.3 Stacked Method Calls

A stack is a well-known mechanism to store the structure necessary to handle chained and (mutually) recursive method calls. In order to describe nested operation calls and, in particular, object recursion[3], a control stack is indispensable. Object recursion is a common principle in object orientation and provides much flexibility and expressiveness. Almost all design patterns (see e.g. [4]) as well as callback-mechanisms of frameworks (e.g. [3]) rely on this principle.

Thus, the information needed in order for computation to resume after a method has finished is standardly store in a stack. The notion of stack used is abstract, the information stored in the stack is organized in frames which include, among others, program counter values.

Although using abstractions, the matter is complicated enough to justify an incremental definition of the method call mechanism. Firstly the single-threaded case is considered.

A stack frame, introduced in Definition 4.4.5, contains the relevant information about the method in execution. A frame on top refers to a method executing at the moment, or to be started now. A frame below the top of the stack refers to a method executing at the moment that has passed control and is blocked. The relevant information in a frame includes the object identifier to which the method belongs, the name of the method, the current program counter value of the method, the object identifier of the calling object, and the (current) variable assignment for formal parameters and local variables of the method. *StackFrame* defines the minimal information needed for the description of stack frames; additional conditions can be added to further constrain stack frames.

[3]That is, a method calls another method of the same object. In contrast, method recursion is when a method which has not finished execution yet is called from a method of an other object.

Definition 4.4.5 (Stack frames)

StackFrame

$$FRAME = UOID \times UOMNAME \times VarAssign \times UPC \times UOID$$
$$framesOf : UMETH \to \mathbb{P}(FRAME)$$

$$framesOf(m) = \{\ (callee, nameOf(m), val, pc, caller) \mid \exists\, op \in UOPN : m = impl(op) \wedge callee \in oids(classOf(op)) \wedge pc \in pcOf(m) \wedge val \in parOf(m) \oplus localsOf(m) \}$$

FRAME is the universe of frames;
framesOf is the set of possible frames for a given method.

Derived:
$$framesOf(m) = \bigcup_{op \in UOPN, m = impl(op)} oids(classOf(op)) \times \{nameOf(m)\} \times (parOf(m) \oplus localsOf(m)) \times pcOf(m)$$

In the case of a single-threaded system, the only existing thread can be defined as an element of type $Stack(FRAME)$.

The method may fork several control flows. Nevertheless, frames have only one program counter. When a fork takes place, a new thread is started. Each thread is then represented by its own stack of frames, each of which contains again only one program counter. Therefore, the definition of frames suffices also for multi-threaded case, the only difference is that there are more than one stack of such frames.

4.4.4 Multiple-thread Computation, Centralized View

The concurrency concept of the system model is orthogonal to objects, i.e., various concurrent threads may independently and even simultaneously "enter" the same object. In the following, a model of threads is added to the system model definition introduced so far. The increment is general enough to allow specialization to other approaches, and is based on an assumed notion of atomic action (whose precise definition is deferred to the defintion of UML actions).

In Definition 4.4.6, the (abstract) universe of possibly infinitely many threads is introduced. The control store maps a stack of frames to each thread. These stacks satisfy the following condition: for any two adjacent frames in the stack, the calling object above is the called object below. An illustration of the central control store with concurrently executing threads is given in Example 4.4.7.

4.4.5 Multiple-thread Computation, Object-Centric View

The central control store defined above is rather general, but so far does not cover how concurrent threads are executed within an object. To enable a general mechanism

Definition 4.4.6 (The control store in centralized version)

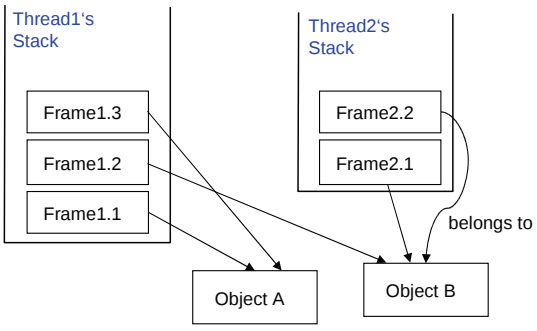
Thread

$UTHREAD$
$CentralControlStore \subseteq (UTHREAD \rightarrow Stack(FRAME))$

$\forall\, ccs \in CentralControlStore, t \in UTHREAD :$
$\quad \forall\, n < \#ccs(t) : \exists\, oid \in UOID :$
$\quad\quad ccs(t)[n] = (oid, *, *, *, *) \wedge ccs(t)[n+1] = (*, *, *, *, oid)$

UTHREAD is the universe of threads.
CentralControlStore assigns a stack of frames to each thread.

Example 4.4.7 (Centralized view on concurrently executing threads) The figure below illustrates the situation where two threads are active, and both object recursion as well as concurrency occurs. Here "Frame$x.y$" denotes that the frame is in thread x at position y, where the highest y-numbers denote the active frames:



Definition 4.4.8 (The control store in object-centric version)

ControlStore

$$ControlStore \subseteq (UOID \to UTHREAD \to Stack(FRAME))$$
$$. \sim . \subseteq CentralControlStore \times ControlStore$$

$$ccs \sim cs \Leftrightarrow$$
$$\forall\, oid \in UOID, t \in UTHREAD :$$
$$cs(oid)(t) = filter(\{(oid, *, *, *, *)\}, ccs(t))$$

ControlStore splits each stack in parts that belong to objects.
. $\sim$. relates two representations of the control store by essentially filtering the centralized stack with regards to individual objects.

Lemma 4.4.9 (Control store representations are equivalent)

$$\forall\, ccs \in CentralControlStore : \exists^1\, cs \in ControlStore : ccs \sim cs$$
$$\forall\, cs \in ControlStore : \exists^1\, ccs \in CentralControlStore : ccs \sim cs$$

for scheduling and definition of priorities, the representation of thread-based stacks is rearranged by providing a different view on threads. The key idea is to use an object-centric view of stacks instead of the current thread-centric view as shown in Definition 4.4.8. As an important side effect, objects are then described in a self-contained way. This means that the control information in the system and the object state in full provide a compositional view on object-oriented systems.

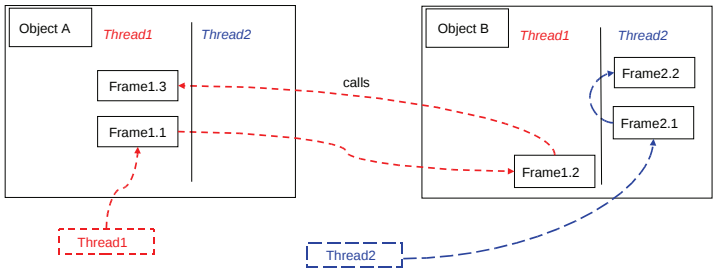
For a *ControlStore cs* the stack $cs(oid)(t)$ contains exactly those frames where a method from object *oid* was called in thread *t*.

Note that the relation . $\sim$. defines an isomorphism as formulated in Lemma 4.4.9. The decentralization into a control store is by definition a function. However, the original stacks can also be uniquely reconstructed because the caller object identifier is part of the frame of the called object. So both representations of the control store provide exactly the same information differently arranged.

Example 4.4.10 shows the above Example 4.4.7 as represented by the object-centered control store.

According to the definition of method calls below (cf. 4.5.3 below), an object can easily recognize that it is being called a second time within the same thread. This is important when, for instance, scheduling or blocking incoming messages from other threads. The Java synchronization model distinguishes recursive calls from other threads and calls from the same threads, and blocks the former but not the latter.

Example 4.4.10 (Object-centric view on concurrently executing threads)



4.5 MESSAGES AND EVENTS IN THE SYSTEM MODEL

In this Section, messages and events are specified as well as how they are stored and handled within objects.

4.5.1 Messages, Events and the Event Store

A uniform handling of events and messages is allowed when messages are considered events as well, and this gives rise to a general concept likewise called "Events". Events can be handled by an operation being executed, or a blocked operation continues execution (in case of a return event), or ignored. They need not be consumed in the order they appear, a more sophisticated management (scheduling) can be individually defined for each object: event occurrences may be handled immediately, or their handling may be delayed until it is made possible, or they can even be ignored.

To capture this rather general notion, a universe of events occurring in systems is introduced. Events are not further structured yet; below certain kinds of events, like method call and return, are introduced as special forms of events. Further specializations are left open.

In Definition 4.5.1, the universe *UEVENT* of events and the universe *EventStore* of event stores are introduced. An event store buffers events that have occurred and are waiting for being processed. A buffer is a rather general structure to store and handle messages, deal with priorities, etc. Event occurrences are instances of events that may store information like the time the instance occurred and possibly other state information. In the system model, hence, event occurrences correspond to system states in which the event has just been added (sent) or removed (received) from the event store.

The following Definition 4.5.2 introduces the universe *UMESSAGE* of messages and the event kind *MsgEvent*. Messages are a general mechanism to encode any kind of synchronous method call, as well as asynchronous message passing. Each message has a unique sender and a unique receiver. That is, a direct description of broadcasting or multicasting is not possible. This means no restriction though: multicasting, for example, can be simulated by repeatedly sending the same message

Definition 4.5.1 (EventStore and object event signature)

EventStore

$$UEVENT$$
$$eventsIn : UOID \to \mathbb{P}(UEVENT)$$
$$eventsOut : UOID \to \mathbb{P}(UEVENT)$$
$$EventStore \subseteq (UOID \to Buffer(UEVENT))$$

$$events(oid) = eventsIn(oid) \cup eventsOut(oid)$$
$$\forall\, es \in EventStore : es(oid) \in Buffer(events(oid))$$

UEVENT is the universe of events.
eventIn are the events that an object may receive.
eventOut are the events that an object may generate.
EventStore maps an object identifier to a buffer of processable events.

Definition 4.5.2 (Object message signature)

Message

$$UMESSAGE$$
$$MsgEvent : UMESSAGE \to UEVENT$$
$$sender, receiver : UMESSAGE \to UOID$$
$$msgIn, msgOut : UOID \to \mathbb{P}(UMESSAGE)$$

$$\forall\, m \in UMESSAGE, oid \in UOID :$$
$$sender(m) = oid \Leftrightarrow MsgEvent(m) \in eventsOut(oid)$$
$$receiver(m) = oid \Leftrightarrow MsgEvent(m) \in eventsIn(oid)$$
$$msgIn(oid) = \{m \mid receiver(m) = oid\}$$
$$msgOut(oid) = \{m \mid sender(m) = oid\}$$

UMESSAGE is the universe of messages.
MsgEvent wraps messages into events that can then be adequately stored. *sender* and *receiver* enforce uniqueness of sender and receiver, respectively, of any message.

to different addressees. Moreover, no further distinction between the various possible forms of messages is enforced.

4.5.2 Method Call and Return Messages

Common kinds of messages describe method call and return. The well-known technique of encoding method call and return into messages, as practiced in distributed systems, among other things supports “remote procedure calls”.

Call messages carry the usual information, like caller and called objects, method name, parameter values, thread. All possible invocations for given caller and called

Definition 4.5.3 (Method call messages)

MethodCall

$$callsOf : UOID \times UOPN \times UOID \times UTHREAD \to \mathbb{P}(UMESSAGE)$$
$$callsOf : UOID \to \mathbb{P}(UMESSAGE)$$

$$\forall r, s \in UOID, op \in UOPN, th \in UTHREAD :$$
$$callsOf(r, op, s, th) \subseteq UOID \times UOMNAME \times TUPLE \times UOID \times UTHREAD$$
$$callsOf(r, op, s, th) = \{(r, nameOf(op), pars, s, th) \mid$$
$$r \in oids(classOf(op)) \wedge$$
$$pars \in params(op)\}$$
$$callsOf(r, op, s, th) \subseteq msgIn(r)$$
$$callsOf(r, op, s, th) \subseteq msgOut(s)$$

$$callsOf(r) = \bigcup_{op \in UOP, s \in UOID, th \in UTHREAD} callsOf(r, op, s, th)$$

callsOf defines the set of all possible method calls from object s to r with operation signature op and run in thread th.

Definition 4.5.4 (Return messages)

MethodReturn

$$returnsOf : UOID \times UOPN \times UOID \times UTHREAD \to \mathbb{P}(UMESSAGE)$$
$$returnsOf : UOID \to \mathbb{P}(UMESSAGE)$$

$$\forall r, s \in UOID, op \in UOPN, th \in UTHREAD :$$
$$returnsOf(r, op, s, th) \subseteq UOID \times UVAL \times UOID \times UTHREAD$$
$$returnsOf(r, op, s, th) = \{(r, v, s, th) \mid$$
$$s \in oids(classOf(op)) \wedge v \in CAR(resType(op))\}$$
$$returnsOf(r, op, s, th) \subseteq msgIn(r)$$
$$returnsOf(r, op, s, th) \subseteq msgOut(s)$$

$$returnsOf(r) = \bigcup_{op \in UOP, s \in UOID, th \in UTHREAD} returnsOf(r, op, s, th)$$

returnsOf defines the set of all possible returns from object s to r that may occur as response to a method call in thread th.

object, operation, and thread, are packed by the function *callsOf* into an appropriate message; see Definition 4.5.3.

Return messages carry the return value, the thread, the sender and receiver of the result value. So the Definition 4.5.4 only slightly differs from the previous definition of method calls. According to the definition of *returnsOf*, the receiver r of the return message was the sender of the original method call.

The concepts of method calls and returns, on the one hand, and of messages, on the other, can be gathered into one single concept of message passing. Message passing allows the handling of composition of objects and provides a clear interface definition

Definition 4.5.5 (Signals as asynchronous messages)

Signal

$$USIGNAL \subseteq UMESSAGE$$

$$callsOf(*,*,*,*) \cap USIGNAL = \varnothing$$
$$returnsOf(*,*,*,*) \cap USIGNAL = \varnothing$$

for objects and object groups. Method calls and returns are then just special kinds of messages and can be treated together with other kinds of incoming messages.

4.5.3 Asynchronous Messages

Formally, signals are just asynchronous messages that do not transfer any control. Therefore, not every message needs to carry a thread marker. There moreover may exist signals that an object may accept. In this case, the object needs to be "active" in the sense that it already has an internal thread to process the stimulus. It is furthermore possible that the object is not in itself active, but belongs to a group of objects that has a common scheduling concept for the processing of messages that come from outside the group. This concept resembles the situation in classical language realizations where one process contains many objects. A concept of regions allows the description of such a common scheduling strategy.

In Definition 4.5.5 the universe *USIGNAL* of signals is introduced, which is a subset of the universe of messages.

4.6 OBJECT STATE

Objects may have an individual state, and groups of objects may have a collective state.

4.6.1 Individual Object States

The signature and the state space of an object comprises data, control and event stores. The three stores are defined as mappings from *UOID* to the respective state elements. Thus, the state of an object is fully described by a value of *OSTATE* as given in Definition 4.6.1.

4.6.2 Grouped Object States

The functions *state* and *states*, introduced in Definition 4.6.2, can be generalized to define the actual and potential set of states for groups of objects. These generalizations use a mapping from object identifiers to their respective contents and are thus structurally equivalent to *STATE*. The structural equivalence of *STATE* and $states(os)$ raises the possibility to use a composition on object states in Lemma 4.6.3 that can be

Definition 4.6.1 (State space of an individual object)

*ObjectStates*1

$$STATE \subseteq DataStore \times ControlStore \times EventStore$$
$$oids : STATE \to \mathbb{P}(UOID)$$
$$OSTATE = INSTANCE \times (UTHREAD \to Stack(FRAME)) \times Buffer(UEVENT)$$
$$state : STATE \times UOID \to OSTATE$$
$$states : UOID \to \mathbb{P}(OSTATE)$$

$$STATE = \{(ds, cs, es) \mid dom(ds) = dom(cs) = dom(es)\}$$
$$oids(ds, cs, es) = oids(ds) = dom(ds)$$
$$\forall\, oid \in oids(us) : state((ds, cs, es), oid) = (ds(oid), cs(oid), es(oid))$$
$$states(oid) = \{state(us, oid) \mid us \in STATE \wedge oid \in oids(us)\}$$

The state of an object consists of its actual attribute values, the events and the threads belonging to an object. *states* defines the potential states of an object.

Derived:

$$oids(ds, cs, es) = dom(ds) = dom(cs) = dom(es)$$

furthermore used to compose state machines in the following section. In particular, $f \oplus g$ is well defined, as $state(us, os_i)$ is equal on the common objects $os_1 \cup os_2$. This also allows to regard the possible set of object states in *states* as a cross product, where the common object identifiers need to coincide in their state.

Definition 4.6.2 identifies $states(o)$ and $states(\{o\})$ as equivalent, as the latter is a function with a singleton domain only.

4.7 EVENT-BASED OBJECT BEHAVIOUR

Based on the notions of state for each object and the corresponding incoming and outgoing events, the behaviour of an object is specified in form of a state transition system. For this purpose the theory of state transition systems (STS) as defined in Appendix A.1 is used.

An STS-based representation of basic actions is required. For that purpose an ordinary programming language such as Java is used. The special actions of UML (cf. [8, Chap. 11]) were disregarded because of the better expressiveness of Java.

4.7.1 Control Flow State Transition Systems

As objects react to incoming events, an STS describing object behaviour is basically event-based and does not necessarily describe timing aspects. To trigger the next execution step for thread *th* within an object, a pseudo-event $\dagger(th)$ is used as given in

Definition 4.6.2 (State space of sets of objects)

ObjectStates2

$state : STATE \times \mathbb{P}(UOID) \to (UOID \to OSTATE)$
$states : \mathbb{P}(UOID) \to \mathbb{P}(UOID \to OSTATE)$

$\forall os \subseteq UOID, us \in STATE, oid \in UOID :$
$\quad state(us, os)(oid) = state(us, oid)$
$\forall os \subseteq UOID :$
$\quad states(os) = \{state(us, os) \mid us \in STATE \wedge os \subseteq oids(us)\}$

Function *state* and *states* can be generalized to define the actual and potential set of states for groups of objects.

Derived:

$\forall os \subseteq UOID, us \in STATE : dom(state(us, os)) = os \cap dom(us)$
$\forall os \subseteq UOID, f \in states(os) : dom(f) = os \cap dom(us)$

Lemma 4.6.3 (State space composition)

ObjectStates

$\forall os_1, os_2 \subseteq UOID, us \in STATE :$
$\quad state(us, os_1 \cup os_2) = state(us, os_1) \oplus state(us, os_2)$
$\forall os_1, os_2 \subseteq UOID, os_1 \cap os_2 = \varnothing \Rightarrow$
$\quad states(os_1 \cup os_2) = \{f_1 \oplus f_2 \mid f_i \in states(os_i), i = 1, 2\}$

Function *state* and *states* are compositional wrt. the state of objects.

Derived:

$\forall os, os_1, os_2 \subseteq UOID : os = os_1 \cap os_2 \Rightarrow$
$states(os_1 \cup os_2)$
$= \{f_1 \oplus (f_2 \mid_{os_2 \setminus os_1}) \mid f_i \in states(os_i)\}$
$= \{(f_1 \mid_{os_1 \setminus os_2}) \oplus f_2 \mid f_i \in states(os_i)\}$
$= \{(f_1 \oplus f_2) \mid f_i \in states(os_i) \wedge f_1 \mid_{os} = f_2 \mid_{os}\}$

Definition 4.7.1 (The stepper for an STS)

STSStepper

$\dagger : UTHREAD \rightarrow STEP$

$injective(\dagger)$

$\dagger(th)$ is used as trigger for the next execution step in thread *th*.

Variation Point 4.7.2 (Control flow STS for methods)

$[CFSTS]$

$cfsts : UMETH \times UOID \times UTHREAD \rightharpoonup STS(S, I, O)$

$\forall\, m \in UMETH, oid \in UOID, th \in UTHREAD :$
$classOf(oid)\ sub\ classOf(m) \wedge cfsts(m, oid, th) = (S, I, O, \delta, s0) \Rightarrow$
$S = \{(o, fr) \in objects(oid) \times framesOf(m) \mid$
$fr = (oid, *, *, *, *)\} \wedge$
$s0 = \{(o, fr) \in S \mid \exists\, start \in StartPC : fr = (*, *, *, start, *)\} \wedge$
$I = \{MsgEvent\ call \mid call \in callsOf(oid, m, *, th)\} \cup STEP \wedge$
$O = eventsOut(oid)$

cfsts assigns a possibly underspecified CFSTS to each method. This describes the implemented behaviour of that method in form of a state machine.

Definition 4.7.1. With this trigger as explicit input of an STS, the scheduling can be defined in a separate entity.

Transitions within the control flow STS (CFSTS) are regarded as atomic actions. A CFSTS is defined in such a way that an object has no direct access to an attribute of any other object, but may call methods and send events as desired. The state of a CFSTS is defined by the objects own attributes and the currently active frame. Variation Point 4.7.2 introduces CFSTS, and uses STS as introduced in Definition A.1.1.

Note that there are alternatives to describe the result of method execution, by e.g. using actions as defined in [8, Chap. 11]. An action language may encompass an ordinary programming language but allow additional actions that deal with manipulation of associations, timing and scheduling, etc.

Indeed, to define such high-level, "model-aware" actions is useful, as otherwise such concepts need to be emulated through lower-level concepts if at all possible. This would mean for instance that associations are encoded as attributes, and scheduling managed through an API of an ordinary object serving as scheduler.[4]

Variation Point 4.7.2 is not constraining CFSTS. Certainly, many states of the CFSTS will never be reached, many outputs that are included in O will not be made.

[4] In Java this would be a Thread object.

Definition 4.7.3 (Event-based STS for objects)

EventSTS

$$ests : UOID \rightarrow STS(S, I, O)$$

$$\forall\, oid \in UOID :$$
$$ests(oid) \in STS(states(oid), eventsIn(oid) \cup STEP, eventsOut(oid))$$

ests assigns a possibly underspecified STS to each *oid*, thus allowing to describe externally visible behaviour for an object as a state machine.

However, it is relatively accurate on the input, as it describes all information about the context that is known.

Note that one CFSTS for each method implementation is attached to each object individually. This gives some freedom, allowing different behaviours for the objects of the same class. In practice, however, objects of one class are assigned the same CFSTS. Furthermore, objects of subclasses whose methods are not overridden are assigned the same CFSTS as their superclass objects. This resembles method inheritance on the level of behaviour through CFSTS.

4.7.2 Event-Based State Transition Systems

Objects react to incoming events and can therefore be described by an STS. This behaviour does not describe timing aspects. Event-based STS (ESTS) handle execution within a single object.

Definition 4.7.3 specifies the general structure and signature of ESTS. An ESTS operates on the full object state and is triggered either by real events or by steps indicated by a †. Those steps denote only scheduling of steps, not timing.

The nondeterministic transition function δ of an ESTS supports underspecification and thus multiple possible behaviours within the STS. This underspecification may be totally or partially resolved during design time by the developer or during runtime by the system itself choosing transitions according to some circumstances, sensor input, etc.

Compared to the previously defined CFSTS, this notion of ESTS is rather general. It embodies all data, control, and event states on a very general level and thus can describe interference of parallel executions as well as handling of incoming events in the buffer. In contrast to an CFSTS, an ESTS embodies the complete object state including control state and event buffer. A detailed description of the relationship between an CFSTS and an ESTS can be found in [1], where also a variation point for ESTS is provided that is composed of several CFSTS.

4.8 TIMED OBJECT BEHAVIOUR

This section presents the time-aware version of STS, called timed STS (TSTS) as defined in Appendix A.2. TSTS allow the description of individual object behaviour and composition thereof.

A discrete global time is assumed available. Each step of transition of the TSTS corresponds to a progress of one time unit. A system executes in steps, each step consumes a fixed amount of time. TSTS are transition systems that deal with this paradigm. Roughly speaking, in each step a finite set of input events is provided to a TSTS, and a finite set of output events is produced by the TSTS.

As a further mechanism, communication channels allow the description of the interaction (communication flow) between parts of the objects and thus of the behaviour of objects on a very fine-grained level.

As a general result, a complete description of how systems are decomposed into objects is provided, as well as what states objects may have, and how objects interact.

4.8.1 Object Behaviour in the System Model

In the system model, the object and component instances cooperate by asynchronous message passing. Method invocation is already modeled by the exchange of two events, the method invocation event and the method return event.

Communication between objects is dealt with by channels. A communication channel is a unidirectional communication connection between two objects. The system model defines a universe UCN of channels and leaves open how many channels are used between objects.

Each channel has a name, e.g., $c \in UCN$, and the type of events that may flow through c is given by $csort(c)$. Each object has a number of incoming and outgoing channels, and each event is associated with the channel through which it flows; see Definition 4.8.1.

As an important consequence of the above definitions, each channel is the in the output signature of only one object, since events are not only associated to a channel but also to its originating object. This ensures the applicability of composition techniques for TSTS, which only work if the output channels of composed objects are disjoint.

Based on channels and their type, the behaviour of a single object is defined in Definition 4.8.2. This definition is based on the assumption of a time granularity fine enough to ensure the independence of the output in one step from the input received in that step. In this way, strong causality between input and output is preserved. The composition of state machines is moreover simplified since feedback within one time unit is ruled out and thus causal inconsistencies are avoided. The actual (real-)time occurrence of events can be abstracted away, thus only the untimed behaviour of objects needs be considered.

Definition 4.8.3 provides a flexible concept of components including, e.g., classical sequential systems (in this case, there is only one input and one output channel). Input and output flow of events can further restricted allowing the reception or the dispatch

Definition 4.8.1 (Channel signatures of objects)

Channels

$$UCN$$
$$sender, receiver : UCN \to UOID$$
$$channel : UEVENT \to UCN$$
$$inC, outC : UOID \to \mathbb{P}(UCN)$$
$$csort : UCN \to \mathbb{P}(UEVENT)$$

$$\forall\, m \in UEVENT, oid \in UOID :$$
$$sender(m) = oid \Rightarrow sender(channel(m)) = oid$$
$$receiver(m) = oid \Rightarrow receiver(channel(m)) = oid$$
$$\forall\, c \in UCN :$$
$$inC(oid) = \{c \mid receiver(c) = oid\}$$
$$outC(oid) = \{c \mid sender(c) = oid\}$$
$$csort(c) = \{m \in UEVENT \mid channel(m) = c\}$$

UCN denotes the universe of channel names.
sender and *receiver* assign a sending and a receiving object to each channel.
channel assigns a channel to each event.
inC, *outC* denote the channel signature of each object.
The type of each channel $csort(c)$ describes the possible events flowing over that channel.

Definition 4.8.2 (Behaviour of individual objects)

ObjBehaviour

$$beh : UOID \to \mathcal{B}^{csort}(I, O)$$

$$\forall\, oid \in UOID :$$
$$beh(oid) \in \mathcal{B}^{csort}(inC(oid), outC(oid))$$

$beh(oid)$ denotes the behaviour of one single object.

Definition 4.8.3 (Behaviour of object compositions)

CompBehaviour

$beh : \mathbb{P}(UOID) \rightarrow \mathcal{B}^{csort}(I, O)$
$inC, outC : \mathbb{P}(UOID) \rightarrow \mathbb{P}(UCN)$

$\forall\, os \subset UOID :$
$I = inC(os) = \{c \mid receiver(c) \in os \wedge sender(c) \notin os\}$
$O = outC(os) = \{c \mid sender(c) \in os \wedge receiver(c) \notin os\}$
$beh(os) = \bigoplus_{oid \in os} beh(oid)$

$beh(os)$ denotes the behaviour of a group of objects where internal communication is not visible anymore.
inC describe the incoming channels for a group of objects, *outC* the outgoing channels.

Definition 4.8.4 (Behaviour as TimedSTS)

TimedSTS

$tsts : UOID \rightarrow TSTS^{csort}(S_1, I_1, O_1)$
$tsts : \mathbb{P}(UOID) \rightarrow TSTS^{csort}(S, I, O)$

$\forall\, oid \in UOID :$
$tsts(oid) \in TSTS^{csort}(states(oid), inC(oid), outC(oid))$
$\mathbb{S}(tsts(oid)) = beh(oid)$
$\forall\, os \subset UOID :$
$tsts(os) = \bigoplus_{oid \in os} tsts(oid)$

$tsts(oid)$ denotes the TSTS-based description of behaviour of one single object. The definition is then generalized to a set of objects.

of at most one event in each step. At the other extreme, highly concurrent systems with a large number of input and output events in one state transition step can likewise be described.

4.8.2 State-based Object Behaviour

The behaviour of an object *oid* is precisely defined as $beh(oid)$. The relationship of this behaviour to a state-based view is not defined yet. For this purpose, a timed state transition system to each object is attached as shown in Definition 4.8.4. According to this definition, each object $oid \in UOID$ can be described by a nondeterministic TSTS as introduced in Appendix A.2.

The axiom $\mathbb{S}(tsts(oid)) = beh(oid)$ for any *oid* states that the behaviour of each object is defined by an appropriate TSTS. It can be shown that the composition of TSTS and of I/O-behaviours is compatible. This means, it can be switched

between a state-based and a purely I/O-based view of object behaviour, and moreover the behaviour of objects or groups (components) can be individually specified and afterwards meaningfully composed.

Note that each object *oid* has exactly one single TSTS $tsts(oid)$. However, as $tsts(oid)$ is a nondeterministic state machine, it allows various forms of underspecification. Therefore, there is no need to add a further concept of underspecification by, e.g., assigning a set of possible TSTS to each object. Any UML model, however, may have an impact on the elements of a TSTS. For instance, the sets of reachable states can be constrained, the initial states restricted to be a singleton, or the nondeterminism reduced by enforcing a behaviour that is deterministic in reaction and time.

With this last part of the system model, a TSTS for the whole system is available that includes all snapshots and all system states and is thus capable of describing any behavioural and structural restrictions by $tsts(UOID)$. The overall system $tsts(UOID)$ does not have any external channels, it incorporates all "objects". This also includes objects that have direct connections to interfaces to other systems, mechanical devices or users and thus can act as surrogates for the context of the system. In other words, the overall system makes a closed worlds assumption. In [9] includes a discussion on how to deal with a closed world assumption to describe open, reactive systems, and also what the advantages are implied by this assumption. In [1], also a general mapping from event-based to TSTS is defined.

4.9 THE SYSTEM MODEL DEFINITION

The last Definition 4.9.1 finally introduces the universe of system models.

REFERENCES

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Definition 4.9.1 (Definition of the system model as a universe)

SYSMOD

SYSMOD

$sm \in SYSMOD \Rightarrow$
$sm =$
$(UTYPE, UVAL, CAR,$
$UVAR, vtype, vsort,$
$UCLASS, UOID, attr, oids, classof,$
$sub, \&,$
$UASSOC, classes, extraVals, relOf,$
$UOPN, UOMNAME, nameOf, classof, parTypes, params, resType,$
$UMETH, UPC, nameof, definedIn, parNames,$
$localNames, resType, pcOf, impl,$
$UTHREAD,$
$UVENT, eventIn, eventsOut,$
$UMESSAGE, MsgEvent, USIGNAL,$
$ests,$
$UCN,$
$tsts)$
such that all constraints defined above are fulfilled.

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Appendix

A.1 STATE TRANSITION SYSTEMS

As objects react on incoming events, state transition systems are an appropriate way of describing object behaviour. Several forms of state transition systems and their compositions are used in the theory. Therefore, we introduce the basics of STS as a general technique here.

A.1.1 STS-Definition

The theory used here is based on the theory of automata, but was partly enhanced in [9] to describe a form of automata, called I/O^*-automata, where transitions are

Definition A.1.1 (I/O^*-STS)

STS

$$
\begin{aligned}
&STS(S, I, O) = \\
&\quad \{(S, I, O, \delta, s0) \mid s0 \subseteq S \wedge s0 \neq \varnothing \\
&\qquad \wedge\, \delta \in S \times I \to \mathbb{P}(S \times O^*) \\
&\qquad \wedge\, \forall s \in S, i \in I : \delta(s, i) \neq \varnothing\}
\end{aligned}
$$

Notation:
$\delta : s \xrightarrow{i/o} t$ is shorthand for $(o, t) \in \delta(s, i)$

$STS(S, I, O)$ is the set of all, possibly underspecified STS with given state, input and output sets. An STS has a complete transition relation as $\delta(s, i) \neq \varnothing$ for all s, i.

triggered by one incoming event and the effect of this event, namely a sequence of possible outputs is the output of the same transition. In contrast to I/O-automata [6], this form allows to abstract away from many internal states of the automaton, which are necessary, if each output is triggered by an individual transition. The application of I/O^*-automata to our description of objects is given in Definition A.1.1.

As can be seen from the definition the transition function is nondeterministic. This allows to model underspecification and thus multiple behaviours in the STS. As discussed in [9], this underspecification may be resolved during design time by the developer or during runtime by the system itself taking the choice according to some random circumstances, sensor input, etc.

The semantics of such an STS is defined in [9] using stream processing functions in the form of [2]. These stream processing functions allow composition, behavioural refinement etc.

However, STS themselves are not fully compositional regarding compositionality of the state space. But there are quite a number of techniques to combine smaller STS to a larger STS.

A.2 TIMED STATE TRANSITION SYSTEMS

Timed state transition systems do not directly use events to make their steps, but time progress. A timed state machine equidistantly performs its steps as time progresses and consumes all events arriving at that time. As a big advantage, we cannot only integrate time into the specification technique, but also have composition operators at hand that are compatible with the composition on streams.

A.2.1 Definition of Timed State Transition Systems

A timed state transition system (TSTS) is a STS where each transition resembles a time step. Such a time step can handle several input events and produce several